Dynamic analysis of the reaction chamber for the ELIADE array

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Abstract The ELI-NP Array of DEtectors (ELIADE) is one of the experimental setups being built at ELI-NP. The reaction chamber for the ELIADE array is, together with the CCD camera the goal of research team. The precision of the experiment make important the small deformation and the vibration of the equipment. To study this, a model using Finite Element Method is used. The device consists of elastic elements to a smaller or greater extent. Usually, if the velocities and the occurring loads are low then the rigid elements hypothesis can lead to an excellent model. But in the experiment that will be conduct in the laboratory with ELIADE, precision is so important that it require extremely small deformation of the device. For this a model that takes into account the elasticity of the body must be used in order to study the movement of the part of the device during the experiment.

1. Introduction

The Gamma Beam System (GBS) is based on the creation of high energy photons after the collision between a visible light laser (2.3eV, 515 nm) and a high energy electron beam. The collision between the free electrons (not bound "inside" an atom) and photons is described by a process named "Compton scattering". In nature, this kind of collisions usually takes place between an electron at rest and a high energy photon, and results in the electron gaining energy at the expense of the

photon which loses some of its energy. Because in the GBS, the roles are reversed (the electron has much more energy than the photon), the photon is the one gaining energy, and as a result, we say that we are dealing with "an inverse Compton scattering" , [1]

The ELI-NP Array of DEtectors (ELIADE) is one of the experimental setups being built at ELI-NP which could benefit from the present project. From the physics point of view the array is made of 8 (up to 12) HyperPure Germanium (HPGe) segmented clover detectors. The experiments envisaged for ELI-NP require this detectors to be placed on two rings, relative to the forward direction of the gamma beam. The detector's axis are all converging to a "theoretical" point which we call "the center of the array". Ideally, this is point where the photon beam intersects the target inside the reaction chamber (Fig.1).

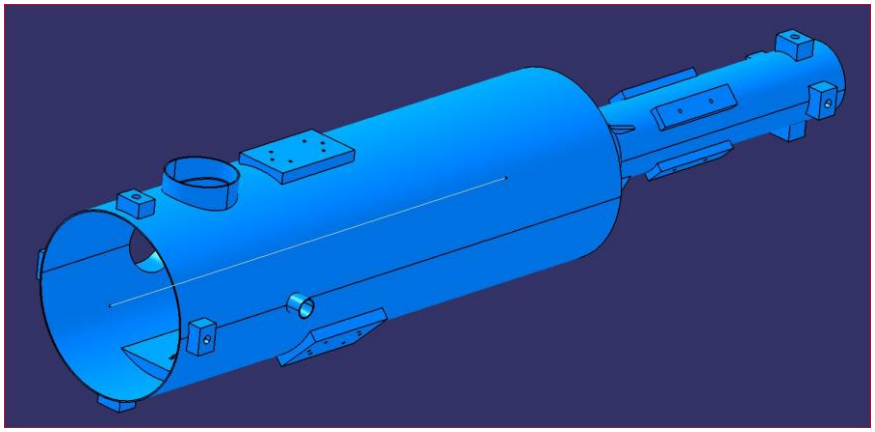

Fig. 1. The sketch of the interaction chamber

An important distinction that needs to be done is the difference between the "center of array" (described above) and the "desired center of the target". The surface of the gamma beam spot size on the target depends on a number of factors, but will typically range between 0.5 and 2 mm in diameter. Ideally, the two are identical, but deviations from the ideal case have very different impacts on the experiments. The problem becomes the precision obtains in the experiment. To achieve this is necessary to make an suitable model in order to obtain a dynamical response of the system.

2. Current state and research context

The hypothesis of rigid body, frequent used in the dynamical analysis of multibody systems, may be satisfactory in most applications. But the different part of a device consists of elastic elements to a smaller or greater extent. If the precision of the experiment imposes to know the deformations, that must be small, the elasticity becomes a significant element. In our case the deformations and the possible vibration may have, generally, an unfavorable influence on the operation of the device.

Continuous mathematical models can be applied, from theoretical point of view, but are not useful in practical applications. The best way of approaching the problem is to apply the finite element method. The advantages of this approach result from [2]-[6].

The papers approaching this field have performed an analysis of a single deformable element, having a plane motion and then the study extended to the mechanisms with plane-parallel motion [7],[8] with all the deformable elements. In [9] the results obtained in this field are being synthesized and some theoretical assumption are presented in [10]-[14].

3. FEM model

The type of finite element used shall determine the chosen shape functions and the equations of motion shall be determined in a general case without considering certain shape functions. In what follows it is considered that deformations are small not influencing the general rigid motion of the whole system. It is considered that a chosen arbitrary finite element together with the solid a component of which it actually is, participates to the entire rigid motion of the mechanical system. The velocities and accelerations of the points of this finite element shall be entirely determined if the velocity and acceleration of the origin of coordinate system the finite element is related to, the angular velocity and angular acceleration are known. In order to write the equations of motion for the studied finite element the equations of Lagrange shall be used. The kinetic energy and the strain energy for the finite element and the work of the distributed and concentrated forces shall be also determined. If these values have been determined we can write the Lagrange function and apply the equations of Lagrange in order to be able to determine the equations of motion of the nodal points. The bodies forming the mechanical system will be considered as being linear elastic.

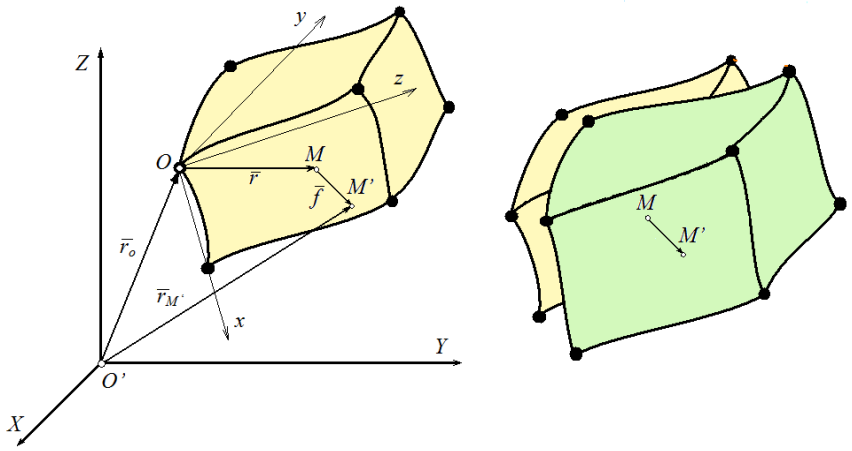

Fig. 2. A representative finite element

Let's consider a finite element belonging to an elastic element of the mechanical system studied. The finite element considered shall be related to the local mobile coordinate system, which participates to the general rigid motion (Fig.2). The multi-body system consisting of several solids, these vectors will be different for each solid composing the system. The transformation of a vector from the local system of coordinates into the global system of coordinates occurs by means of a matrix ***R.***

If we note the position vector of point M with $\boldsymbol{r}_{M,G}$ we may write:

$$\boldsymbol{r}_{M,G} = \boldsymbol{r}_{O,G} + \boldsymbol{r}_{G} = \boldsymbol{r}_{O,G} + \boldsymbol{R} \cdot \boldsymbol{r}_{L} \quad , \tag{1}$$

where index G indicates a vector with the components expressed in the global coordinate system and index L a vector with the components expressed in the local coordinate system.

If point M has a displacement $\boldsymbol{f}_{L}$ changing into M', we may write:

$$\boldsymbol{r}_{M',G} = \boldsymbol{r}_{O,G} + \boldsymbol{R} \cdot (\boldsymbol{r}_{L} + \boldsymbol{f}_{L}) \tag{2}$$

where $\boldsymbol{r}_{M',G}$ is the position vector of point *M*' with components expressed in the global coordinate system. The continuous displacement field $\boldsymbol{f}(x,y,z)_{L}$ is ap-

proximated in the finite element method, depending on the nodal coordinates, by relation:

$$\boldsymbol{f}_{L} = \boldsymbol{N}(x,y,z)\ \boldsymbol{\delta}_{e}(t)_{L} \tag{3}$$

where the matrix elements $\boldsymbol{N}$ - the shape functions -, will depend on the type of the finite element chosen.

4. Equation of motion for a three-dimensional finite element

The equations of motion shall be obtained in the local coordinate system. For this purpose the equations of Lagrange shall be used [15]. The kinetic energy of the considered finite element is given by the expression:

$$E_c = \frac{1}{2}\int_V \rho v^2 dV = \frac{1}{2}\int_V \rho\ \boldsymbol{v}^{T}_{M',G}\ \boldsymbol{v}_{M',G}\, dV. \tag{4}$$

The potential energy (internal work) is:

$$E_p = \frac{1}{2}\int_V \boldsymbol{\sigma}^{T}\ \boldsymbol{\varepsilon}\, dV. \tag{5}$$

We remind you of the Hooke law which we write as follows:

$$\boldsymbol{\sigma} = \boldsymbol{D}\ \boldsymbol{\varepsilon} \tag{6}$$

for an homogeneous, isotropic material. The differential relations which link the strains to the finite deformations that can be expressed in a concise form:

$$\boldsymbol{\varepsilon} = a\ \boldsymbol{f} \tag{7}$$

where $\boldsymbol{a}$ represents the differentiation operator (see [6]). If using the relations (6) and (7) the strains energy results:

$$E_p = \frac{1}{2}\int_V \boldsymbol{\delta}^{T}_{e,L}\ \boldsymbol{k}_{e}\ \boldsymbol{\delta}_{e,L}\, dV \quad , \tag{8}$$

where $\boldsymbol{k}_{e}$ is the stiffness matrix:

$$\boldsymbol{k}_{e} = \int_V \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{a}^{\boldsymbol{T}} \boldsymbol{D}^{\boldsymbol{T}} \boldsymbol{a}\, \boldsymbol{N}\ dV. \tag{9}$$

If the distributed forces vector is noted with $\boldsymbol{p} = \boldsymbol{p}(x,y,z)$, then the external work is:

$$W = \int_V \boldsymbol{p}_L^{\,T} \boldsymbol{f}_L\, dV = \left(\int_V \boldsymbol{p}_L^{\,T} \boldsymbol{N}\ dV\right) \boldsymbol{\delta}_{e,L}\,. \tag{10}$$

The nodal forces $\boldsymbol{q}_e^{\,T}$ give an external work:

$$\boldsymbol{W}^{c} = \boldsymbol{q}_{e,L}^{\,T} \boldsymbol{\delta}_{e,L}\ . \tag{11}$$

The Lagrangean for the considered element will be:

$$L = E_c - E_p + W + W^c\ . \tag{12}$$

The equations of motion are obtained by applying the equations of Lagrange. After a series of elementary calculations and rearranging of terms we get the equations of motion for the finite element considered:

$$\left(\int_V \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{N}\ \rho dV\right) \ddot{\boldsymbol{\delta}}_{e,L} + 2\left(\int_V \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{R}^{\boldsymbol{T}} \dot{\boldsymbol{R}}\, \boldsymbol{N}\ \rho dV\right)$$

$$\dot{\boldsymbol{\delta}}_{e,L} + \left(\boldsymbol{k}_{e} + \int_V \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{R}^{\boldsymbol{T}} \ddot{\boldsymbol{R}}\, \boldsymbol{N}\ \rho dV\right) \boldsymbol{\delta}_{e,L} =$$

$$= \boldsymbol{q}_{e} + \int_V \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{p}_L\, dV - \left(\int_V \boldsymbol{N}^{\boldsymbol{T}} \rho dV\right) \boldsymbol{R}^{\boldsymbol{T}}\ \ddot{\boldsymbol{r}}_{o} - \int_V \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{R}^{\boldsymbol{T}} \ddot{\boldsymbol{R}}\, \boldsymbol{r}\ \rho dV \tag{13}$$

The equations of motion can be written in a condensed form:

$$\boldsymbol{m}_{e} \ddot{\boldsymbol{\delta}}_{e,L} + 2\boldsymbol{c}_{e}\ \dot{\boldsymbol{\delta}}_{e,L} + \left[\boldsymbol{k}_{e} + \boldsymbol{k}_{e}(\varepsilon) + \boldsymbol{k}_{e}(\omega^2)\right] \boldsymbol{\delta}_{e,L} =$$

$$= \boldsymbol{q}_{e} + \boldsymbol{q}_{e,L}^{\,*} - \boldsymbol{q}_{\boldsymbol{e},\boldsymbol{L}}^{\,\boldsymbol{i}}(\varepsilon) - \boldsymbol{q}_{e,L}^{\,i}(\omega^2) - \boldsymbol{m}_{\boldsymbol{o}\boldsymbol{e}}^{\,\boldsymbol{i}}\, \boldsymbol{R}^{T}\, \ddot{\boldsymbol{r}}_{o}\ . \tag{14}$$

The equations of motion are related to the local system of coordinates and can be linearized the system of reference being considered as " frozen" in that particular position, in which the field of velocities and accelerations is known. The matrix coefficients can be calculated after choosing the shape functions and the nodal coordinates for expressing the displacement of a point.

The assembling operation leads to the elimination of the liaison forces (see [14]-[16]). The involved matrices can be computed by choosing the nodal coordinates and the shape functions for the chosen finite element.

5. Conclusions

The obtained equations in the study of the dynamic response have additional terms. The first additional term is $2\boldsymbol{c}_e\,\dot{\boldsymbol{\delta}}_{e,L}$ and is due to the relative motion of nodal coordinates relative to the mobile coordinate systems attached to the moving bodies - Coriolis effects. The second additional term is $\boldsymbol{k}_e(\varepsilon) + \boldsymbol{k}_e(\omega^2)$ and represents a change in stiffness determined by the accelerations field of relative motion. These two terms can become significant in the dynamic response of the multibody system and can change not only quantitative but also qualitative this response. In order to obtain a very high accuracy in the motion and control of the interaction chamber the presented model was used in the project of the reaction chamber for ELIADE array.

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